A New Fibonacci-Lucas Relation

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Very recently, B. Sury [1] proved an interesting and new Fibonacci-Lucas relation, namely,

\[ 2^{m+1}F_{m+1} = \sum_{i=0}^{m} 2^i L_i. \]

In his proof, Sury used a simple polynomial identity (see [2] for another proof). Naturally, a question arises: is there a similar formula for \( 3^{m+1}F_{m+1} \)? In this note, we shall provide such a formula. More precisely

**Theorem.** For all \( m \geq 1 \), it holds that

\[
3^{m+1}F_{m+1} = \sum_{i=0}^{m} 3^i L_i + \sum_{i=0}^{m+1} 3^{i-1} F_i.
\]

**Proof.** We can proceed by induction on \( m \). Clearly, the basis case is straightforward, so we may suppose that the identity holds for \( m \in \{k-1, k\} \). Now, we sum these relations and by adding

\[
3^k L_k + 2 \cdot 3^{k+1} L_{k+1} + 3^k F_{k+1} + 2 \cdot 3^{k+1} F_{k+3}
\]
in each side, we arrive at

\[
2 \left( \sum_{i=0}^{k+1} 3^i L_i + \sum_{i=0}^{k+2} 3^{i-1} F_i \right) = 3^k \left( L_k + 6F_{k+1} + 6L_{k+1} + 6F_{k+2} \right),
\]

where we used that \( L_i = F_{i-1} + F_{i+1} \).

**References**


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